

# Infrared cutoff dependence of the critical flavor number in three-dimensional QED

V. P. Gusynin\*

*Department of Applied Mathematics, University of Western Ontario, London, Ontario, Canada N6A 5B7*

M. Reenders

*Department of Polymer Chemistry and Materials Science Center, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands*

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We solve, analytically and numerically, a gap equation in parity invariant QED<sub>3</sub> in the presence of an infrared cutoff  $\mu$  and derive an expression for the critical fermion number  $N_c$  as a function of  $\mu$ . We argue that this dependence of  $N_c$  on the infrared scale might solve the discrepancy between continuum Schwinger-Dyson equations studies and lattice simulations of QED<sub>3</sub>.

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Parity invariant quantum electrodynamics in 2+1 dimensions with  $N$  flavors of four-component massless fermions (QED<sub>3</sub>) [1] has been attracting a lot of interest for almost two decades. While it was often regarded as a nice polygon for studying nonperturbative phenomena in gauge field theories, such as dynamical mass generation, recently the model has found applications in condensed matter physics, in particular in high- $T_c$  superconductivity [2].

QED<sub>3</sub> is ultraviolet finite and has a built-in intrinsic mass scale given by the dimensionful gauge coupling  $e$ , or  $\alpha = e^2 N/8$ , which plays a role similar to the  $\Lambda$  scale parameter in QCD. In the leading order in  $1/N$  expansion, it was found that at large momenta ( $p \gg \alpha$ ) the effective coupling between fermions and gauge bosons vanishes (asymptotic freedom) whereas it has a finite value or infrared stable fixed point at  $p \ll \alpha$  [3]. This behavior was shown to be robust against the introduction of higher order  $1/N$  corrections [4,5]. Studies of the gap equation for a fermion dynamical mass in the leading order in the  $1/N$  expansion have shown that massless QED<sub>3</sub> exhibits chiral symmetry breaking whenever the number of fermion species  $N$  is less than some critical value  $N_c$ , which is estimated to be in the region  $3 < N_c < 5$  ( $N_c = 32/\pi^2 \approx 3.2$  in the simplest ladder approximation [6]). A renormalization group analysis gives  $3 < N_c < 4$  [7]. Below such a critical  $N_c$ , the  $U(2N)$  flavor symmetry is broken down to  $U(N) \times U(N)$ , the fermions acquire a dynamical mass, and  $2N^2$  Goldstone bosons appear; for  $N > N_c$  the particle spectrum of the model consists of interacting massless fermions and a photon. In addition, it was argued that the dynamical symmetry breaking phase transition at  $N = N_c$  is a conformal phase transition [8,9]. The last one is characterized by a scaling function for the dynamical fermion mass with an essential singularity [6,8].

On the other hand, there is an argument due to Appelquist *et al.* [10] that  $N_c \leq 3/2$ . The argument is based on the inequality  $f_{\text{IR}} \leq f_{\text{UV}}$  where  $f$  is the thermodynamic free energy which is estimated in both infrared and ultraviolet regimes by counting massless degrees of freedom.

The above-described version of QED<sub>3</sub>, the so-called parity invariant noncompact version, was studied on a lattice [11]. Recent lattice simulations of QED<sub>3</sub> with  $N \geq 2$  have found no decisive signal for chiral symmetry breaking [12]. In particular, for  $N = 2$ , Ref. [12] reports an upper bound for the chiral condensate  $\langle \bar{\psi}\psi \rangle$  to be of order  $5 \times 10^{-5}$  in units of  $e^4$ . We recall that for quenched,  $N = 0$ , QED<sub>3</sub> both numerical [13] and analytical [14] studies have shown that chiral symmetry is always broken and numerically  $\langle \bar{\psi}\psi \rangle \sim 5 \times 10^{-3}$ . The above result of Ref. [12] seemingly contradicts studies based on Schwinger-Dyson (SD) equations which advocate dynamical symmetry breaking for  $N = 2$  and seems to favor the estimate of Appelquist *et al.* (we mention also that work in progress is being done on lattice simulations of QED<sub>3</sub> with a single,  $N = 1$ , fermion flavor [15]).

One of the major problems in studying dynamical symmetry breaking using lattice simulations is to obtain control of finite size effects, which play a nontrivial role due to the presence of a massless photon [12]. In terms of the intrinsic lattice spacing  $a$ , three dimensionless length scales appear in lattice simulations: the size of the lattice  $L \sim 10-50$ , the dimensionless lattice coupling constant  $\beta \propto 1/(e^2 a)$ , and the bare fermion mass  $m_0 a$ . In order to establish dynamical symmetry breaking the role of all these scales in the problem should be well under control. At present, lattice discretization appears to be well understood; however, the finite size effects appear nontrivial and are most likely the source of the discrepancy between lattice and continuum studies of QED<sub>3</sub> [12].

In this paper, we present a possible explanation for the discrepancy between recent lattice and continuum studies of QED<sub>3</sub> by studying analytically and numerically the gap equation in the presence of the infrared (IR) cutoff  $\mu$ , where  $\mu$  is inversely related to the size  $L$  of a lattice. Since the characteristic ratio  $e^2/\mu$  for continuum QED<sub>3</sub> turns out to be proportional to the ratio  $L/\beta$  in lattice simulations [12], a comparison between two approaches can be made. We will show that the presence of an IR cutoff reduces the value of the critical number  $N_c$  and derive the relationship between  $N_c$  and  $\mu$ . Recently, a similar gap equation, but with a massive photon, was studied numerically in Refs. [16,17] in con-

\*On leave of absence from Bogolyubov Institute for Theoretical Physics, 03143, Kiev, Ukraine.

nection with applications of QED<sub>3</sub> to high- $T_c$  superconductors (for an earlier numerical study in QED<sub>3</sub>, see Ref. [18]).

The gap equation of massless QED<sub>3</sub> with IR cutoff  $\mu$  reads [compare with Eq. (3.3) of Ref. [3]]

$$\Sigma(p) = \frac{\lambda\alpha}{2p} \int_{\mu}^{\infty} dk \frac{k\Sigma(k)}{k^2 + \Sigma^2(k)} \ln \frac{k+p+\alpha}{|k-p|+\alpha}, \quad (1)$$

where  $\lambda = 8/N\pi^2$  and the Landau gauge is used. Equation (1) is the simplest approximation to the SD equation for the fermion self-energy which neglects corrections to the fermion wave-function renormalization and vertex corrections. Extensive studies showed that the nature of chiral symmetry breaking which emerges from Eq. (1) with  $\mu=0$  remains qualitatively unchanged after including higher order corrections. In what follows we solve Eq. (1) numerically but in order to be able to treat it analytically one needs to make further approximations. Because the integrand is damped at large momenta of integration, the main contribution comes from the region with momenta  $k \ll \alpha$ ; thus, expanding the logarithm we come to the simplified gap equation

$$\Sigma(p) = \frac{\lambda}{p} \int_{\mu}^{\alpha} dk \frac{k\Sigma(k)}{k^2 + \Sigma^2(k)} \min(k, p). \quad (2)$$

The scale  $\mu$  can be identified with the inverse size of the lattice in the temporal direction as  $\mu \approx \pi/La$  [19] since both scales  $\mu$  and  $\pi/La$  represent a minimal fermion momentum in continuum and lattice theories, respectively [ $\mu$  is related to the photon mass  $m_a$  ( $\mu = m_a^2/\alpha$ ) in the model of Ref. [17]]. Although this argument for the identification of  $\mu$  and  $\pi/La$  (up to a factor of order 1) is certainly heuristic, it is not unreasonable.

On the other hand, the ultraviolet (UV) cutoff  $\alpha$  in Eq. (2) is in fact a physical scale which separates a nonperturbative dynamics at  $k \ll \alpha$  from a perturbative one, with momenta much larger than  $\alpha$ , where the dimensionless running coupling  $\bar{\alpha}(k) = \alpha/(k + \alpha)$  [3] is weak. Neglecting those perturbative contributions in Eq. (1) results in an effective UV cutoff  $\alpha$  in Eq. (2).

We shall solve the above equation (2) using the well-established linearized approximation when the momentum-dependent dynamical mass in the denominator  $k^2 + \Sigma^2(k)$  is replaced by the constant dynamical mass at the lower limit:  $\Sigma^2(k) \rightarrow \Sigma_0^2$ ,  $\Sigma_0 = \Sigma(\mu)$ . This approximation is known to work well, especially near the phase transition point where  $\Sigma_0 \ll \alpha$ . As we show, the linearized equation (2) with an IR cutoff can be dealt with analytically and an exact expression for  $N_c$  as a function of  $\alpha/\mu$  can be derived, following the method developed in Refs. [20–22].

In the linearized approximation, Eq. (2) can be reduced to a hypergeometric differential equation

$$u(1-u)\Sigma''(u) + \frac{3}{2}(1-u)\Sigma'(u) - \frac{\lambda}{4}\Sigma(u) = 0, \quad (3)$$

with the IR and UV boundary conditions

$$\left[ p^2 \frac{d\Sigma}{dp} \right]_{p=\mu} = 0, \quad \left[ p \frac{d\Sigma}{dp} + \Sigma \right]_{p=\alpha} = 0, \quad (4)$$

where  $u = -p^2/\Sigma_0^2$ . The general solution of Eq. (3) can be written as

$$\frac{\Sigma(p)}{\Sigma_0} = c_1 u_1(p) + c_2 u_2(p), \quad (5)$$

where we choose

$$u_1(p) = F\left(\frac{1+i\nu}{4}, \frac{1-i\nu}{4}; \frac{3}{2}; -\frac{p^2}{\Sigma_0^2}\right), \quad (6)$$

$$u_2(p) = \left(\frac{p}{\Sigma_0}\right)^{-(1+i\nu)/2} \times F\left(-\frac{1-i\nu}{4}, \frac{1+i\nu}{4}; 1 + \frac{i\nu}{2}; -\frac{\Sigma_0^2}{p^2}\right) + \text{c.c.}, \quad (7)$$

as a particular pair of independent solutions of Eq. (3), with  $\nu = \sqrt{4\lambda - 1}$ , and where  $F$  is the hypergeometric function. If the infrared scale is set equal to zero ( $\mu=0$ ), then the IR boundary condition gives the constraint  $c_2=0$ , since the function  $u_2$  is too singular at  $p=0$ . Then, the UV boundary condition fixes the relationship between the dynamical mass  $\Sigma_0$ ,  $\nu$ , and  $\alpha$ . For nonzero  $\mu$ , the boundary conditions (4) determine the mass spectrum, giving rise to the equation

$$f(\Sigma_0/\alpha, \mu/\alpha, \nu) = A_1 B_2 - A_2 B_1 = 0, \quad (8)$$

where

$$A_i \equiv \left[ p \frac{du_i}{dp} + u_i \right]_{p=\alpha}, \quad B_i \equiv \left[ p^2 \frac{du_i}{dp} \right]_{p=\mu}. \quad (9)$$

By making use of the formulas for differentiating the hypergeometric function [23] the real functions  $A_i$  and  $B_i$  are expressed as

$$A_1 = F\left(\frac{1+i\nu}{4}, \frac{1-i\nu}{4}; \frac{1}{2}; -\frac{\alpha^2}{\Sigma_0^2}\right), \quad (10)$$

$$A_2 = \left(\frac{\alpha}{\Sigma_0}\right)^{-(1+i\nu)/2} \frac{(1-i\nu)}{2} \times F\left(1 - \frac{1-i\nu}{4}, \frac{1+i\nu}{4}; 1 + \frac{i\nu}{2}; -\frac{\Sigma_0^2}{\alpha^2}\right) + \text{c.c.} \quad (11)$$

and

$$B_1 = -\mu \frac{\mu^2 (1+\nu^2)}{\Sigma_0^2} \frac{1}{12} F\left(1 + \frac{1+i\nu}{4}, 1 + \frac{1-i\nu}{4}; \frac{5}{2}; -\frac{\mu^2}{\Sigma_0^2}\right), \quad (12)$$

$$\begin{aligned}
B_2 &= -\mu \left( \frac{\mu}{\Sigma_0} \right)^{-(1+i\nu)/2} \frac{(1+i\nu)}{2} \\
&\times F \left( 1 + \frac{1+i\nu}{4}, -\frac{1-i\nu}{4}; 1 + \frac{i\nu}{2}; -\frac{\Sigma_0^2}{\mu^2} \right) + \text{c.c.} \\
&= -\mu \left( \frac{\mu}{\Sigma_0} \right)^{-1/2} \sqrt{1+\nu^2} \{ \text{Re}[F(\nu)] \cos \beta_2 \\
&\quad + \text{Im}[F(\nu)] \sin \beta_2 \}, \tag{13}
\end{aligned}$$

where  $\beta_2 = (\nu/2) \ln(\mu/\Sigma_0) - \tan^{-1} \nu$ , and  $F(\nu)$  is shorthand notation for the hypergeometric function:

$$F(\nu) = F \left( 1 + \frac{1+i\nu}{4}, -\frac{1-i\nu}{4}; 1 + \frac{i\nu}{2}; -\frac{\Sigma_0^2}{\mu^2} \right). \tag{14}$$

Since we study the critical behavior, we can always assume that  $\Sigma_0 \ll \alpha$ , and therefore we can use the asymptotic expressions for  $A_1$  and  $A_2$ :

$$\begin{aligned}
A_1 &\approx \left( \frac{\alpha}{\Sigma_0} \right)^{-1/2} \left[ \frac{\Gamma(1/2)\Gamma(i\nu/2)}{\Gamma^2((1+i\nu)/4)} \left( \frac{\alpha}{\Sigma_0} \right)^{i\nu/2} + \text{c.c.} \right] \\
&= \left( \frac{\alpha}{\Sigma_0} \right)^{-1/2} |c(\nu)| \sqrt{1+\nu^2} \cos(\alpha_2 + \theta) \tag{15}
\end{aligned}$$

and

$$A_2 \approx \left( \frac{\alpha}{\Sigma_0} \right)^{-1/2} \sqrt{1+\nu^2} \cos \alpha_2, \tag{16}$$

where

$$\alpha_2 = \frac{\nu}{2} \ln \frac{\alpha}{\Sigma_0} + \tan^{-1} \nu, \quad \theta = \arg c(\nu),$$

$$c(\nu) = \frac{\Gamma(3/2)\Gamma(i\nu/2)}{\Gamma((1+i\nu)/4)\Gamma((5+i\nu)/4)}.$$

By making use of (2.10.2) of [23], we can write  $B_1$  in a form similar to  $B_2$ :

$$\begin{aligned}
B_1 &= -\mu \left( \frac{\mu}{\Sigma_0} \right)^{-(1+i\nu)/2} \frac{(1+i\nu)}{2} c(-\nu) \\
&\times F \left( 1 + \frac{1+i\nu}{4}, -\frac{1-i\nu}{4}; 1 + \frac{i\nu}{2}; -\frac{\Sigma_0^2}{\mu^2} \right) + \text{c.c.}, \\
&= -\mu \left( \frac{\mu}{\Sigma_0} \right)^{-1/2} \sqrt{1+\nu^2} |c(\nu)| \{ \text{Re}[F(\nu)] \cos(\beta_2 + \theta) \\
&\quad + \text{Im}[F(\nu)] \sin(\beta_2 + \theta) \}. \tag{17}
\end{aligned}$$

Finally, using the above expressions (12)–(17), the gap equation (8) can be expressed as

$$\begin{aligned}
f &= -\mu |c| (1+\nu^2) \left( \frac{\alpha\mu}{\Sigma_0^2} \right)^{-1/2} \sin \theta \{ \text{Re}[F(\nu)] \sin(\beta_2 - \alpha_2) \\
&\quad - \text{Im}[F(\nu)] \cos(\beta_2 - \alpha_2) \}. \tag{18}
\end{aligned}$$

One can convince oneself that for  $\mu \ll \Sigma_0$  the last equation is reduced to

$$\cos \left( \frac{\nu}{2} \ln \frac{\alpha}{\Sigma_0} + \theta + \tan^{-1} \nu \right) = 0, \tag{19}$$

which gives the well-known result for the dynamical mass exhibiting the essential singularity at  $\nu \rightarrow 0$ :

$$\Sigma_0 = \alpha \exp \left[ -\frac{2(\pi n + \pi/2 - \theta - \tan^{-1} \nu)}{\nu} \right]. \tag{20}$$

We recall that only the solution with  $n=1$  corresponds to the stable ground state. Note also that for  $N$  close to  $N_c$  ( $\nu \approx 0$ ) the dynamically generated mass  $\Sigma_0$  is much less than the scale  $\alpha$ , providing a hierarchy of mass scales in the model under consideration.

On the other hand, for  $\Sigma_0 \ll \mu$ , we can use the power expansion of  $F(\nu)$  to get the equation for the dynamical mass  $\Sigma_0$ . By expanding  $F(\nu)$  of Eq. (14) for  $\Sigma_0 \ll \mu$ , we obtain

$$\text{Re}[F(\nu)] = 1 + \frac{\Sigma_0^2}{\mu^2} \rho(\nu^2) \cos \phi, \tag{21}$$

$$\text{Im}[F(\nu)] = \frac{\Sigma_0^2}{\mu^2} \rho(\nu^2) \sin \phi, \tag{22}$$

where

$$\rho(\nu^2) = \frac{1}{8} \sqrt{\frac{(25+\nu^2)(1+\nu^2)}{4+\nu^2}}, \tag{23}$$

$$\phi = -\tan^{-1} \frac{\nu(13+\nu^2)}{10-2\nu^2}. \tag{24}$$

Thus Eq. (18) reduces to

$$\frac{\Sigma_0^2}{\mu^2} = -\frac{1}{\rho(\nu^2)} \frac{\sin \left( \frac{\nu}{2} \ln \frac{\alpha}{\mu} + 2 \tan^{-1} \nu \right)}{\sin \left( \frac{\nu}{2} \ln \frac{\alpha}{\mu} + 2 \tan^{-1} \nu + \phi \right)}. \tag{25}$$

Since, for  $0 < \nu < 1$ ,  $\phi$  is negative, we find that a nontrivial or real solution for  $\Sigma_0$  arises when  $\nu$  exceeds the critical value  $\nu_c$  determined by the equation

$$\frac{\nu_c}{2} \ln \frac{\alpha}{\mu} + 2 \tan^{-1} \nu_c = \pi, \tag{26}$$

so that the sin in the numerator of Eq. (25) becomes negative (for  $\nu > \nu_c$ ). Note that the form of the gap equation (18) is

different in two regions  $\mu \ll \Sigma_0$  and  $\mu \gg \Sigma_0$ : while in the first one ( $\mu \ll \Sigma_0$ ) we observe oscillations in the mass variable, in the second one ( $\mu \gg \Sigma_0$ ) such oscillations disappear. This is reflected also in the character of the mass dependence on the coupling constant [compare Eqs. (20) and (29) below]. In general, it can be shown that Eq. (18) has  $n$  nontrivial solutions where the number  $n$  is given by

$$n = \left\lfloor \frac{\nu}{2\pi} \ln \frac{\alpha}{\mu} + \frac{2}{\pi} \tan^{-1} \nu \right\rfloor, \quad (27)$$

and the symbol  $[C]$  denotes the integer part of the number  $C$ . For small  $\nu_c$ , Eq. (26) gives

$$\nu_c = \frac{\pi}{2 + (1/2) \ln(\alpha/\mu)}. \quad (28)$$

A similar functional dependence of  $N_c$  was guessed in Ref. [17] by fitting their numerical study of the gap equation with a massive photon.

Near the critical point, we find from Eq. (25) the mean-field scaling law for  $\Sigma_0$ ,

$$\frac{\Sigma_0^2}{\mu^2} = \sigma(\nu - \nu_c), \quad (29)$$

where the positive “nonuniversal” constant of proportionality,  $\sigma$ , is

$$\sigma = \frac{1}{\rho(\nu_c^2)} \left[ \frac{(\pi - 2 \tan^{-1} \nu_c)}{\nu_c} + \frac{2}{\nu_c^2 + 1} \right] \frac{1}{\sin(\pi + \phi_c)}, \quad (30)$$

with  $\phi_c$  the value of  $\phi$  at  $\nu = \nu_c$ .

We now compare the analytically obtained critical value of  $\nu_c$  [Eq. (26)] or  $N_c$  as a function of the ratio  $\pi e^2/\mu$  ( $\alpha = N e^2/8$ ) with the numerical solution of the integral equation (1) which was obtained by using numerical methods described in Ref. [24]. Basically, starting with some initial guess for  $\Sigma(p)$ , the integral equation is iterated using splines on a logarithmic momentum scale, until sufficient precision is reached. Both the analytical solution for Eq. (2) and numerical solution for Eq. (1) are depicted in Fig. 1. The analytical solution of Eq. (2) agrees reasonable well with the numerical solution of Eq. (1) for values of  $N \gtrsim 1$ . However, Eq. (2) [hence Eq. (26)] provides a rather poor description of dynamical symmetry breaking for small values of  $N$  [actually it does not allow quenched limit in contrast to Eq. (1)]. The bending of the curve for the analytic solution for  $0 \leq N \leq 1$  is due to the linear dependence of the effective UV cutoff  $\alpha$  on  $N$ . Furthermore, the numerical solution shows the existence

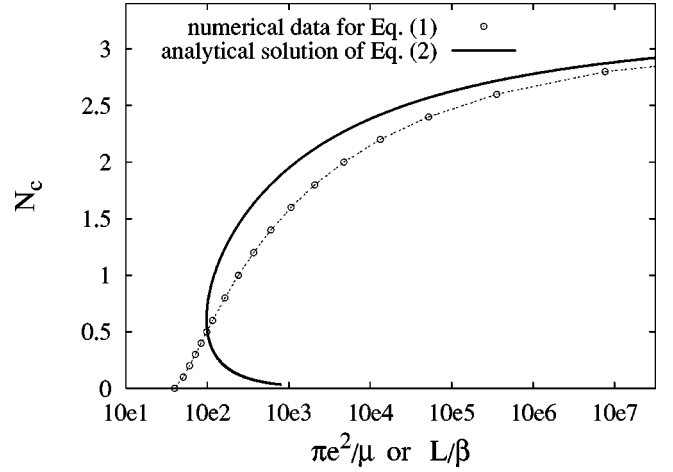


FIG. 1. The numerical solution of Eq. (1) and the analytical solution of Eq. (2) given by Eq. (26) for  $N_c$  versus the ratio  $\pi e^2/\mu$  or the lattice ratio  $L/\beta$ .

of a critical ratio  $\pi e^2/\mu \approx 40$  for  $N=0$  (i.e., quenched QED<sub>3</sub>) below which no chiral symmetry breaking occurs.

From Fig. 1 it can be extracted that in order to find chiral symmetry breaking for  $N=2$  at least a ratio  $\pi e^2/\mu \approx 5 \times 10^3$  is required. Note that in order to draw decisive conclusions on chiral symmetry breaking on a lattice, the volume of the lattice  $L^3$  must be large not just in dimensionless units but also compared to any dynamically generated correlations in the system. In particular, the physical volume  $(L/\beta)^3$  is required to be large [12]. Typical lattice simulations use at the most a ratio  $L/\beta \approx 50$ –100 and appear to lie outside the region for dynamical symmetry breaking. Indeed, by associating  $La \approx \pi/\mu$  and  $e^2 \approx 1/(\beta a)$ , we have  $L/\beta \approx \pi e^2/\mu$ ; thus, dynamical symmetry breaking near  $N=2$  requires a ratio  $L/\beta \approx 5 \times 10^3$ . This explains the absence of any signs of chiral symmetry breaking in lattice simulations with  $N=2$ . For  $N=1$  the ratio is  $L/\beta \approx 240$  for the numerical solution of Eq. (1); therefore, we believe that lattice simulations with a single fermion flavor are crucial in establishing whether there is a critical number  $N_c$  in QED<sub>3</sub> and how this value depends on the size  $L$  of the lattice. Moreover, as is evident from Eq. (29), the scaling near the critical point is expected to be of the mean-field type and such a scaling can also be checked using lattice simulations.

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